

These exercises are from *Fundamental Methods of Mathematical Economics*, 4th edition, by A. C. Chiang and K. Wainwright.

### 4.2.1.

Given  $A = \begin{bmatrix} 7 & -1 \\ 6 & 9 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$ , and  $C = \begin{bmatrix} 8 & 3 \\ 6 & 1 \end{bmatrix}$ , find:

- $A + B$
- $C - A$
- $3A$
- $4B + 2C$

### 4.2.2.

Given  $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$ , and  $C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$ :

- Is  $AB$  defined? Calculate  $AB$ . Can you calculate  $BA$ ? Why?
- Is  $BC$  defined? Calculate  $BC$ . Is  $CB$  defined? If so, calculate  $CB$ . Is it true that  $BC = CB$ ?

### 4.2.4.

Find the product matrices in the following.

a.  $\begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$

b.  $\begin{bmatrix} 6 & 5 & -1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$

c.  $\begin{bmatrix} 3 & 5 & 0 \\ 4 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

d.  $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

### 4.3.1.

Given  $u^T = [5 \ 1 \ 3]$ ,  $v^T = [3 \ 1 \ -1]$ ,  $w^T = [7 \ 5 \ 8]$ , and  $x^T = [x_1 \ x_2 \ x_3]$ , find:

- a.  $uv^T$
- c.  $xx^T$
- d.  $v^T u$

### 4.4.1.

Given  $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 7 \\ 8 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & 4 \\ 1 & 9 \end{bmatrix}$ , verify that

- a.  $(A + B) + C = A + (B + C)$

### 4.4.3.

Test the associative law of multiplication with the following matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -8 & 0 & 7 \\ 1 & 3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 7 & 1 \end{bmatrix}$$

### 4.4.5.

Find  $C = AB$ .

- a.  $A = \begin{bmatrix} 12 & 14 \\ 20 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & 9 \\ 0 & 32 \end{bmatrix}$

### 4.5.1.

Given  $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$ ,  $b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$ , and  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ :

Calculate

1.  $AI$
2.  $IA$
3.  $Ix$
4.  $x^T I$

Indicate the dimension of the identity matrix used in each case.

### 4.5.2.

Using the matrices given in 4.5.1, calculate

- a.  $Ab$
- b.  $AIb$
- c.  $x^T IA$
- d.  $x^T A$

Does the insertion of  $I$  in part b affect the result in part a? Does the insertion of  $I$  in part d affect the result in part c?

### 4.5.3.

Using the matrices given in 4.5.1, what is the dimension of the null matrix resulting from each of the following?

- a. Premultiply  $A$  by a  $5 \times 2$  null matrix.
- b. Postmultiply  $A$  by a  $3 \times 6$  null matrix.

### 4.6.1.

Given  $A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}$ , and  $C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$ , find  $A^T$ ,  $B^T$ ,  $C^T$ .

### 4.6.2.

Use the matrices given in 4.6.1 to verify that

- a.  $(A + B)^T = A^T + B^T$
- b.  $(AC)^T = C^T A^T$